ASSIGNMENT SET – I

Mathematics: Semester-IV

M.Sc (CBCS)

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

PAPER - MTM-405A

Paper: Operational Research Modelling-II

Answer all the questions

1. In a certain community 25% of all girls are blondes and 75% of all blondes have blue eyes. Also 50% of all girls in the community have blue eyes. If you know that a girl has blue eyes, how much additional information do you get by being informed that she is blonde?
2. What is channel matrix? State fundamental theorem of information theory.
3. State and Proof the additive property of entropy function.
4. Obtain reliability over 100 hours period of a system consisting of two subsystems A and B connected in parallel where A consists of 4 identical components in series and B consists of 3 identical components in parallel. Each component has a reliability 0.90 over a period of 100 hours.
5. Show that R (t) = exp $[-\int_0^t \lambda(t)dt]$, where (t) is the reliability function and (t) represents the failure rate. What is mean time between failure ?
6. Find the curve x = (t) which minimizes the function J = $\int_0^1 (\dot{x}^2 + 1)$dt where (0) = 1 and $x(1)$ = 2.
7. Describe the Pontryagin's Maximum Principle.
8. What is memory less channel? Discuss about measure of information.
9. The failure rate of an electronic subsystem is 0.0005 failure/hour. If a operational period of 500 hours with probability of success 0.95 is desired. What label of parallel redundancy is needed?
10. What is joint entropy? Prove that $H(X,Y) \leq H(X) + H(Y)$ with equality iff X and Y are independent.

11. Obtain reliability over 100 hours period of a system consisting of two subsystems A and B connected in parallel where A consists of 4 identical components in series and B consists of 3 identical components in parallel. Each component has reliability 0.90 over a period of 100 hours.
12. Show that the entropy of the following probability distribution of the events $x_1, x_2, \ldots x_i \ldots x_{n-1}, x_n$ with probability $\frac{1}{2}, \frac{1}{2^2}, \ldots \frac{1}{2^i}, \ldots \frac{1}{2^{n-1}}, \frac{1}{2^{n-1}}$.
13. Describe the Bang Bang control and illustrate it with the help of an example.
14. Consider a binary channel with input symbols A={0, 1}, output symbols B={0, 1} and the channel matrix $\begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{9} & \frac{8}{9} \end{bmatrix}$. The input probabilities $p_{10} = \frac{4}{5}$, $p_{20} = \frac{1}{5}$ Find the conditional backward input probabilities and joint probabilities. State fundamental theorem of information theory.
15. In a system, there are n number of components connected in parallel with reliability $R_i(t)$=n, i=1, 2, ... n. Find the reliability of the system. If $R_1(t) = R_2(t) = \ldots\ldots = R_n(t) = e^{-\lambda t}$. Then find the reliability of the system.
16. Let X_n be a particular event with probability p_n is distributed into m mutually exclusive sub-events $Y_1, Y_2 \ldots Y_m$ with probabilities $q_1, q_2 \ldots q_m$ respectively, such that $p_n = q_1 + q_2 + \ldots + q_m$ then
$$H(p_1, p_2, \ldots p_{n-1}, q_1, q_2, \ldots q_m) = H(p_1, p_2, \ldots p_{n-1}, p_n) + p_n H\left(\frac{q_1}{p_n} + \frac{q_2}{p_n} + \ldots + \frac{q_m}{p_n}\right).$$
17. An electrochemical system is characterized by the ordinary differential equation $\frac{dx_1}{dt} = x_2$ and $\frac{dx_2}{dt} + x_2 = u$ where u is the control variable chosen in such a way that the cost function $\frac{1}{2}\int_0^a (x_1^2 + 4u^2)dt$ is minimized. Show that if the boundary conditions satisfied by the state variables are $x1(0) = a, x2(0) = b$, where a, b are constants and $x_1 \to 0, x_2 \to 0$ as $t \to \infty$, the optimal choice for u is $u = -\frac{1}{2}x_1(t) + (1-\sqrt{2})x_2(t)$.
18. Draw the diagram of a communication system mentioning all the important components including noise system.
19. Define entropy function and explain its importance.

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